Pearson New International Edition

# Calculus for Scientists and Engineers
## Early Transcendentals

W. Briggs L. Cochran B. Gillett

First Edition

ALWAYS LEARNING

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ISBN 10: 1-292-02480-1
ISBN 13: 978-1-292-02480-6

**British Library Cataloguing-in-Publication Data**
A catalogue record for this book is available from the British Library

Printed in the United States of America

# Table of Contents

# Functions



**Chapter Preview** The goal of this chapter is to ensure that you begin your calculus journey fully equipped with the tools you will need. Here, you will see the entire cast of functions needed for calculus, which includes polynomials, rational functions, algebraic functions, exponential and logarithmic functions, and the trigonometric functions, along with their inverses. It is imperative that you work hard to master the ideas in this chapter and refer to it when questions arise.

## 1 Review of Functions

Mathematics is a language with an alphabet, a vocabulary, and many rules. We assume that you are familiar with set notation, intervals on the real number line, absolute value, the Cartesian coordinate system, and equations of lines and circles. This chapter is about the fundamental concept of a function.

Everywhere around us we see relationships among quantities, or **variables**. For example, the consumer price index changes in time and the temperature of the ocean varies with latitude. These relationships can often be expressed by mathematical objects called *functions*. Calculus is the study of functions, and because we use functions to describe the world around us, calculus is a universal language for human inquiry.

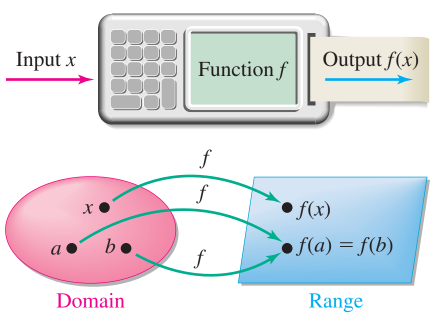


**FIGURE 1**

**DEFINITION Function**

A **function** $f$ is a rule that assigns to each value $x$ in a set $D$ a *unique* value denoted $f(x)$. The set $D$ is the **domain** of the function. The **range** is the set of all values of $f(x)$ produced as $x$ varies over the domain (Figure 1).

➤ If the domain is not specified, we take it to be the set of all values of $x$ for which $f$ is defined. We will see shortly that the domain and range of a function may be restricted by the context of the problem.

The **independent variable** is the variable associated with the domain; the **dependent variable** belongs to the range. The **graph** of a function $f$ is the set of all points $(x, y)$ in the $xy$-plane that satisfy the equation $y = f(x)$. The **argument** of a function is the expression on which the function works. For example, $x$ is the argument when we write $f(x)$. Similarly, 2 is the argument in $f(2)$ and $x^2 + 4$ is the argument in $f(x^2 + 4)$.

**QUICK CHECK 1** If $f(x) = x^2 - 2x$, find $f(-1)$, $f(x^2)$, $f(t)$, and $f(p - 1)$. ◄

The requirement that a function must assign a *unique* value of the dependent variable to each value in the domain is expressed in the vertical line test (Figure 2).


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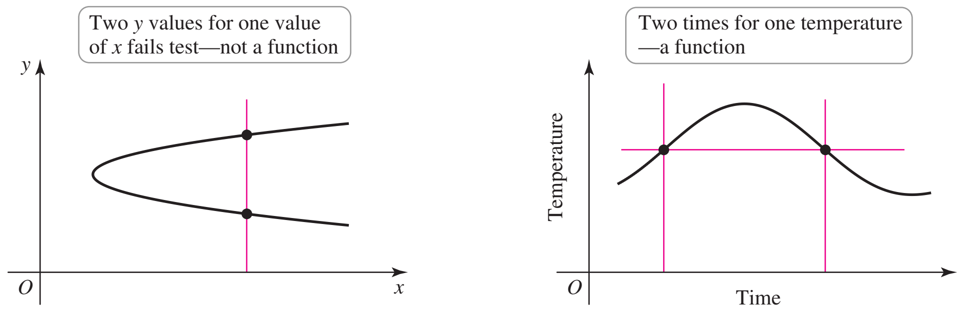


FIGURE 2

> A set of points or a graph that does *not* correspond to a function represents a **relation** between the variables. All functions are relations, but not all relations are functions.

> **Vertical Line Test**
>
> A graph represents a function if and only if it passes the **vertical line test**: Every vertical line intersects the graph at most once. A graph that fails this test does not represent a function.

**EXAMPLE 1** **Identifying functions** State whether each graph in Figure 3 represents a function.

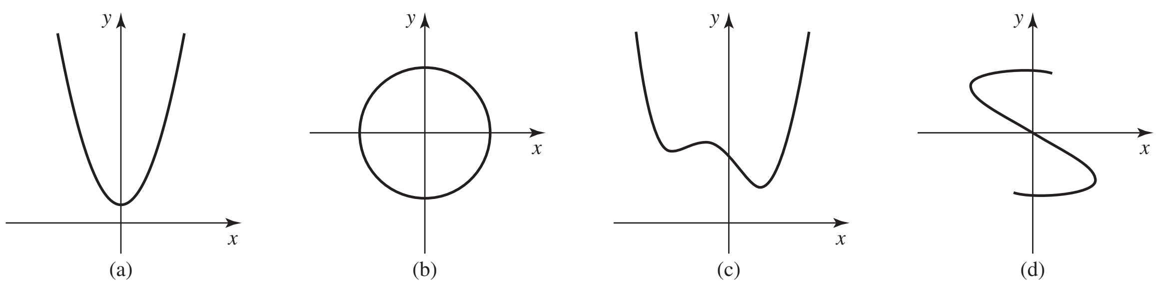


FIGURE 3

**SOLUTION** The vertical line test indicates that only graphs (a) and (c) represent functions. In graphs (b) and (d), it is possible to draw vertical lines that intersect the graph more than once. Equivalently, it is possible to find values of $x$ that correspond to more than one value of $y$. Therefore, graphs (b) and (d) do not pass the vertical line test and do not represent functions.

*Related Exercises 11–12*

> A window of $[a, b] \times [c, d]$ means $a \le x \le b$ and $c \le y \le d$.

**EXAMPLE 2** **Domain and range** Graph each function with a graphing utility using the given window. Then state the domain and range of the function.

**a.** $y = f(x) = x^2 + 1; \quad [-3, 3] \times [-1, 5]$

**b.** $z = g(t) = \sqrt{4 - t^2}; \quad [-3, 3] \times [-1, 3]$

**c.** $w = h = \dfrac{1}{u - 1}; \quad [-3, 5] \times [-4, 4]$

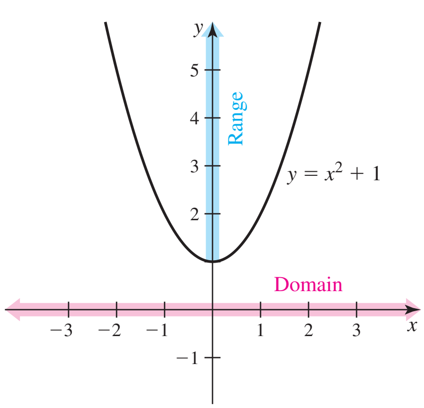


FIGURE 4

**SOLUTION**

**a.** Figure 4 shows the graph of $f(x) = x^2 + 1$. Because $f$ is defined for all values of $x$, its domain is the set of all real numbers, or $(-\infty, \infty)$, or $\mathbb{R}$. Because $x^2 \ge 0$ for all $x$, it follows that $x^2 + 1 \ge 1$ and the range of $f$ is $[1, \infty)$.

**b.** When $n$ is even, functions involving $n$th roots are defined provided the quantity under the root is nonnegative (or in some cases positive). In this case, the function $g$ is defined provided $4 - t^2 \ge 0$, which means $t^2 \le 4$, or $-2 \le t \le 2$. Therefore, the domain of $g$ is $[-2, 2]$. By the definition of the square root, the range consists only of

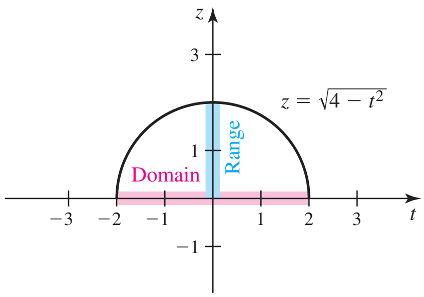


**FIGURE 5**

➤ The dashed vertical line $u = 1$ in Figure 6 indicates that the graph of $w = h(u)$ approaches a *vertical asymptote* as $u$ approaches 1 and that $w$ becomes large in magnitude for $u$ near 1.

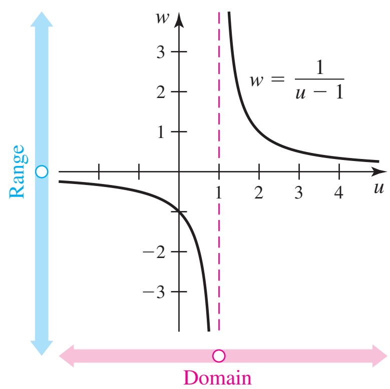


**FIGURE 6**

nonnegative numbers. When $t = 0$, $z$ reaches its maximum value of $g(0) = \sqrt{4} = 2$, and when $t = \pm 2$, $z$ attains its minimum value of $g(\pm 2) = 0$. Therefore, the range of $g$ is $[0, 2]$ (Figure 5).

**c.** The function $h$ is undefined at $u = 1$, so its domain is $\{u: u \neq 1\}$ and the graph does not have a point corresponding to $u = 1$. We see that $w$ takes on all values except 0; therefore, the range is $\{w: w \neq 0\}$. A graphing utility does *not* represent this function accurately if it shows the vertical line $u = 1$ as part of the graph (Figure 6).

*Related Exercises 13–20* ◄

**EXAMPLE 3** **Domain and range in context** At time $t = 0$ a stone is thrown vertically upward from the ground at a speed of 30 m/s. Its height above the ground in meters (neglecting air resistance) is approximated by the function $h = f(t) = 30t - 5t^2$, where $t$ is measured in seconds. Find the domain and range of this function as they apply to this particular problem.

**SOLUTION** Although $f$ is defined for all values of $t$, the only relevant times are between the time the stone is thrown ($t = 0$) and the time it strikes the ground, when $f(t) = 0$. Solving the equation $h = 30t - 5t^2 = 0$, we find that

$$30t - 5t^2 = 0$$

$$5t(6 - t) = 0 \qquad \text{Factor.}$$

$$5t = 0 \quad \text{or} \quad 6 - t = 0 \qquad \text{Set each factor equal to 0.}$$

$$t = 0 \quad \text{or} \quad t = 6. \qquad \text{Solve.}$$

Therefore, the stone leaves the ground at $t = 0$ and returns to the ground at $t = 6$. An appropriate domain that fits the context of this problem is $\{t: 0 \leq t \leq 6\}$. The range consists of all values of $h = 30t - 5t^2$ as $t$ varies over $[0, 6]$. The largest value of $h$ occurs when the stone reaches its highest point at $t = 3$, which is $h = f(3) = 45$. Therefore, the range is $[0, 45]$. These observations are confirmed by the graph of the height function (Figure 7). Note that this graph is *not* the trajectory of the stone; the stone moves vertically.

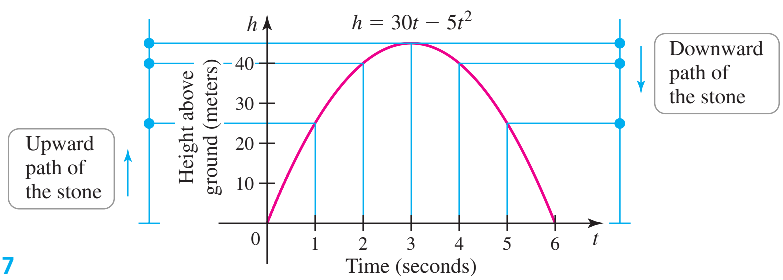


**FIGURE 7**

*Related Exercises 21–24* ◄

**QUICK CHECK 2** What are the domain and range of $f(x) = (x^2 + 1)^{-1}$? ◄

## Composite Functions

➤ In the composition $y = f(g(x))$, $f$ is called the *outer function* and $g$ is the *inner function*.

Functions may be combined using sums $(f + g)$, differences $(f - g)$, products $(fg)$, or quotients $(f/g)$. The process called *composition* also produces new functions.

**DEFINITION** **Composite Functions**

Given two functions $f$ and $g$, the composite function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$. It is evaluated in two steps: $y = f(u)$, where $u = g(x)$. The domain of $f \circ g$ consists of all $x$ in the domain of $g$ such that $u = g(x)$ is in the domain of $f$ (Figure 8).

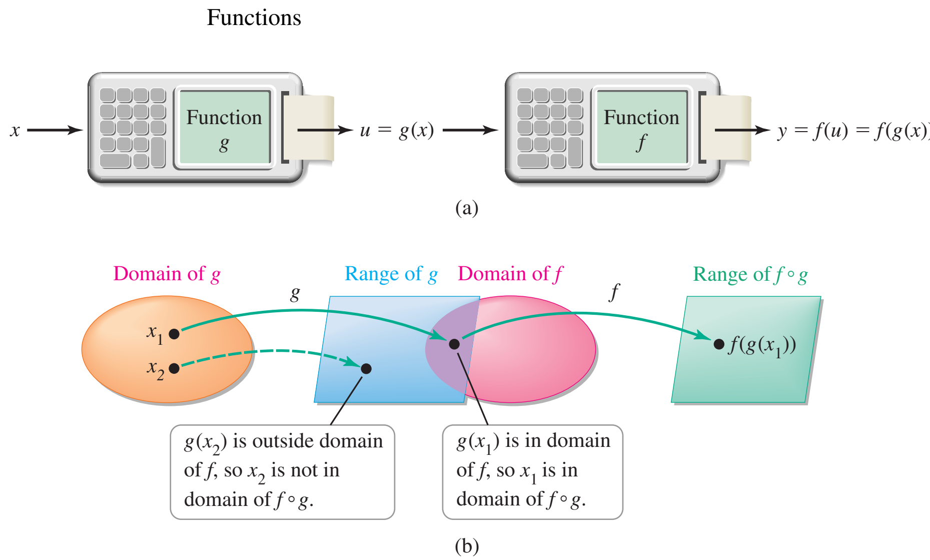


**FIGURE 8**

**EXAMPLE 4** **Composite functions and notation** Let $f(x) = 3x^2 - x$ and $g(x) = 1/x$. Simplify the following expressions.

**a.** $f(5p + 1)$ **b.** $g(1/x)$ **c.** $f(g(x))$ **d.** $g(f(x))$

**SOLUTION** In each case, the functions work on their arguments.

**a.** The argument of $f$ is $5p + 1$, so

$$f(5p + 1) = 3(5p + 1)^2 - (5p + 1) = 75p^2 + 25p + 2.$$

**b.** Because $g$ requires taking the reciprocal of the argument, we take the reciprocal of $1/x$ and find that $g(1/x) = 1/(1/x) = x$.

**c.** The argument of $f$ is $g(x)$, so

$$f(g(x)) = f\left(\frac{1}{x}\right) = 3\left(\frac{1}{x}\right)^2 - \left(\frac{1}{x}\right) = \frac{3 - x}{x^2}.$$

➤ Examples 4c and 4d demonstrate that, in general,

$$f(g(x)) \neq g(f(x)).$$

**d.** The argument of $g$ is $f(x)$, so

$$g(f(x)) = g(3x^2 - x) = \frac{1}{3x^2 - x}.$$

*Related Exercises 25–36* ◂

**EXAMPLE 5** **Working with composite functions** Identify possible choices for the inner and outer functions in the following composite functions. Give the domain of the composite function.

**a.** $h(x) = \sqrt{9x - x^2}$ **b.** $h(x) = \dfrac{2}{(x^2 - 1)^3}$

**SOLUTION**

**a.** An obvious outer function is $f(x) = \sqrt{x}$, which works on the inner function $g(x) = 9x - x^2$. Therefore, $h$ can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of $x$ such that $9x - x^2 \geq 0$. Solving this inequality gives $\{x: 0 \leq x \leq 9\}$ as the domain of $f \circ g$.

**b.** A good choice for an outer function is $f(x) = 2/x^3 = 2x^{-3}$, which works on the inner function $g(x) = x^2 - 1$. Therefore, $h$ can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of $g(x)$ such that $g(x) \neq 0$, which is $\{x: x \neq \pm 1\}$.

*Related Exercises 37–40* ◂

**EXAMPLE 6** **More composite functions** Given $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - x - 6$, find **(a)** $g \circ f$ and **(b)** $g \circ g$, and their domains.

**SOLUTION**

**a.** We have

$$(g \circ f)(x) = g(f(x)) = (\underbrace{\sqrt[3]{x}}_{f(x)})^2 - \underbrace{\sqrt[3]{x}}_{f(x)} - 6 = x^{2/3} - x^{1/3} - 6.$$

Because the domains of $f$ and $g$ are $(-\infty, \infty)$, the domain of $f \circ g$ is also $(-\infty, \infty)$.

**b.** In this case, we have the composition of two polynomials:

$$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= g(x^2 - x - 6) \\ &= \underbrace{(x^2 - x - 6)}_{g(x)}{}^2 - \underbrace{(x^2 - x - 6)}_{g(x)} - 6 \\ &= x^4 - 2x^3 - 12x^2 + 13x + 36.\end{aligned}$$

The domain of the composition of two polynomials is $(-\infty, \infty)$.

*Related Exercises 41–54*◄

**QUICK CHECK 3** If $f(x) = x^2 + 1$ and $g(x) = x^2$, find $f \circ g$ and $g \circ f$.◄

**EXAMPLE 7** **Using graphs to evaluate composite functions** Use the graphs of $f$ and $g$ in Figure 9 to find the following values.

**a.** $f(g(5))$ **b.** $f(g(3))$ **c.** $g(f(3))$ **d.** $f(f(4))$

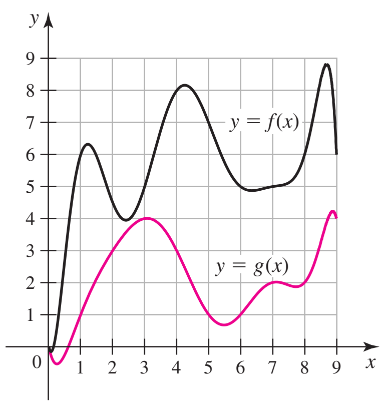


**FIGURE 9**

**SOLUTION**

**a.** According to the graphs, $g(5) = 1$ and $f(1) = 6$; it follows that $f(g(5)) = f(1) = 6$.

**b.** The graphs indicate that $g(3) = 4$ and $f(4) = 8$, so $f(g(3)) = f(4) = 8$.

**c.** We see that $g(f(3)) = g(5) = 1$. Observe that $f(g(3)) \neq g(f(3))$.

**d.** In this case, $f(\underbrace{f(4)}_{8}) = f(8) = 6$.

*Related Exercises 55–56*◄

## Secant Lines and the Difference Quotient

Figure 10 shows two points $P(x, f(x))$ and $Q(x + h, f(x + h))$ on the graph of $y = f(x)$. A line through any two points on a curve is called a **secant line**, and it plays an important role in calculus. The slope of the secant line through $P$ and $Q$, denoted $m_{\text{sec}}$, is given by

$$m_{\text{sec}} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{f(x + h) - f(x)}{h}.$$

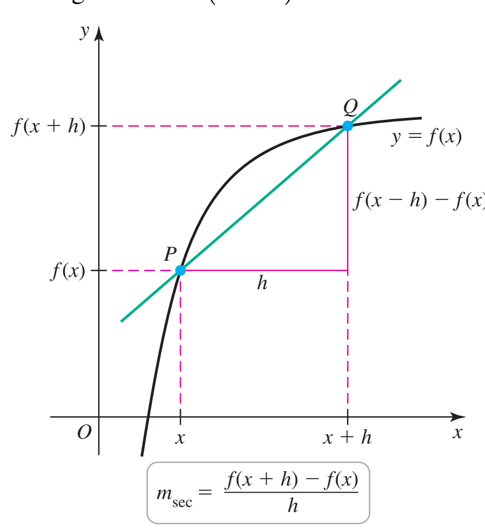


**FIGURE 10**